

Listen

■ Listen im alltäglichen Umgang mit *Mathematica*

```
ClearAll["Global`*"]
```

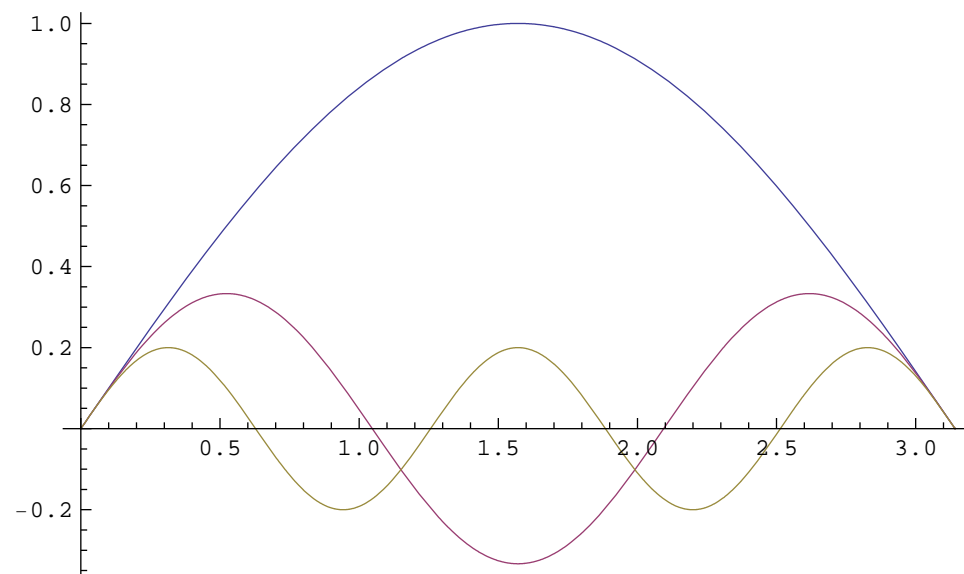
```
Solve[x2 == 1, x]
```

```
{{x → -1}, {x → 1}}
```

```
Solve[{2 x + 4 y == 6, x - y == 0}, {x, y}]
```

```
{{x → 1, y → 1}}
```

```
Plot[{Sin[x],  $\frac{\text{Sin}[3 x]}{3}$ ,  $\frac{\text{Sin}[5 x]}{5}$ }, {x, 0,  $\pi$ }]
```



■ Iteratoren

```
Table[i2, {3}]
```

```
{i2, i2, i2}
```

```
Table[i2, {i, 3}]
```

```
{1, 4, 9}
```

```
Table[i2, {i, 2, 4}]
{4, 9, 16}
```

```
Table[i2, {i, 2, 4,  $\frac{1}{2}$ }]
{4,  $\frac{25}{4}$ , 9,  $\frac{49}{4}$ , 16}
```

```
Table[i2, {i, 4, 2, -1}]
{16, 9, 4}
```

```
Range[5]
{1, 2, 3, 4, 5}
```

```
Range[3, 5]
{3, 4, 5}
```

```
Range[3, 5, 0.3]
{3., 3.3, 3.6, 3.9, 4.2, 4.5, 4.8}
```

■ Listen erzeugen

```
l = {1, 2, 3, 4, 5}
{1, 2, 3, 4, 5}
```

```
l = Table[x2, {x, 0, 4}]
{0, 1, 4, 9, 16}
```

```
Map[#2 &, Range[0, 4]]
{0, 1, 4, 9, 16}
```

```
l = Table[x2, {x, 0, 50, 10}]
{0, 100, 400, 900, 1600, 2500}
```

```
l = Table[xy, {x, 0, 3}, {y, 2, 4}]
{{0, 0, 0}, {1, 1, 1}, {4, 8, 16}, {9, 27, 81}}
```

```
l = Table[Table[xy, {y, 2, 4}], {x, 0, 3}]
{{0, 0, 0}, {1, 1, 1}, {4, 8, 16}, {9, 27, 81}}
```

```
1 // Grid

0 0 0
1 1 1
4 8 16
9 27 81

Table[nm, {n, 1, 3}, {m, 1, n}]

{{1}, {2, 4}, {3, 9, 27}}

Table[Table[nm, {m, 1, n}], {n, 1, 3}]

{{1}, {2, 4}, {3, 9, 27}}

l := Table[f[xn], {n, 1, 5}];
l

{f[x], f[x2], f[x3], f[x4], f[x5] }

f[x_] := x3;
l

{x3, x6, x9, x12, x15 }

Table[x[i], {i, 1, 5}]

{x[1], x[2], x[3], x[4], x[5]}

Array[x, 5]

{x[1], x[2], x[3], x[4], x[5]}

l = {{1, 2}, {3, 4, 5}, {6, 7, 8, 9}};

Length[l]

3

Length[l[[3]]]

4

Length[a + b + c]

3

Depth[{a, b, c}]

2

Depth[l]

3
```

```
{Depth[a + b c], Depth[a + b / c]}  
  
{3, 4}  
  
Depth[{a, b, c + d}]  
  
3
```

■ Auf Listenelemente zugreifen

```
l = {a, b, c, d};  
  
l[[2]]  
  
b  
  
l[[-1]]  
  
d  
  
l[[0]]  
  
List  
  
(a + b)[[0]]  
  
Plus  
  
l[{2, 4}]  
  
{b, d}  
  
Part[l, 2]  
  
b  
  
Part[l, {2, 4}]  
  
{b, d}  
  
Part[l, Range[2, 4]]  
  
{b, c, d}  
  
l[[2 ;; 4]]  
  
{b, c, d}  
  
l = {{a, b, c}, {d, e, f}, {g, h, i}};  
  
l // Grid  
  
a b c  
d e f  
g h i
```

```

1[[1]]
{a, b, c}

1[[1, 3]]
c

Part[1, 1, 3]
c

1[{{2, 3}, {1, 2}}]
{{d, e}, {g, h}}

Part[1, {2, 3}, {1, 2}]
{{d, e}, {g, h}}

l = {1, a, b^c, Sqrt[3]};
{Head[l], First[l], Last[l]}
{List, 1, Sqrt[3]}

Head[a + b]
Plus

```

■ Funktionen und Listen

```

ClearAll["Global`*"]

Map[f, {a, b, c}]
{f[a], f[b], f[c]}

f /@ {a, b, c}
{f[a], f[b], f[c]}

5 {a, b, c}
{5 a, 5 b, 5 c}

Attributes[Plus]
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}

{a, b, c} + {d, e, f}
{a + d, b + e, c + f}

```

```

l = Table[ $\frac{i}{i+1}$ , {i, 5}]

{ $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ ,  $\frac{4}{5}$ ,  $\frac{5}{6}$ }

l // N

{0.5, 0.666667, 0.75, 0.8, 0.833333}

x^{1,2,3,4,5}

{x, x^2, x^3, x^4, x^5}

Apply[f, {a, b, c}]

f[a, b, c]

f@@{a, b, c}

f[a, b, c]

Map[f, a + b + c]

f[a] + f[b] + f[c]

f /@ g[a, b, c]

g[f[a], f[b], f[c]]

Apply[f, a + b + c]

f[a, b, c]

f@@g[a, b, c]

f[a, b, c]

```

■ Listen manipulieren

```

ClearAll["Global`*"]

l = {1, 2, 3, 4};

Append[l, 7]

{1, 2, 3, 4, 7}

l

{1, 2, 3, 4}

Prepend[l, 7]

{7, 1, 2, 3, 4}

```

```

{First[l], Rest[l]}

{1, {2, 3, 4}}

Join[{1, 2, 3, 4}, {6, 4, 5, 3}]

{1, 2, 3, 4, 6, 4, 5, 3}

l = {{{a, b}, {c, d}}, {{e, f}, {g, h}}};

Flatten[l]

{a, b, c, d, e, f, g, h}

Flatten[l, 1]

{{a, b}, {c, d}, {e, f}, {g, h}}

l = Table[Random[Integer, 100], {50}];
Partition[l, 5]

{{84, 80, 9, 32, 99}, {18, 29, 75, 14, 84}, {78, 8, 47, 81, 34},
 {0, 66, 95, 83, 69}, {22, 47, 11, 39, 59}, {62, 42, 91, 80, 66},
 {18, 16, 100, 66, 66}, {93, 48, 89, 27, 49}, {99, 6, 90, 31, 50}, {6, 85, 7, 15, 59}}

l[[#]] & /@ Table[5 i + j, {i, 0, 9}, {j, 1, 5}]

{{84, 80, 9, 32, 99}, {18, 29, 75, 14, 84}, {78, 8, 47, 81, 34},
 {0, 66, 95, 83, 69}, {22, 47, 11, 39, 59}, {62, 42, 91, 80, 66},
 {18, 16, 100, 66, 66}, {93, 48, 89, 27, 49}, {99, 6, 90, 31, 50}, {6, 85, 7, 15, 59}}

l = {{a, b, c}, {d, e, f}}

{{a, b, c}, {d, e, f}}

Thread[l]

{{a, d}, {b, e}, {c, f}}

Transpose[l]

{{a, d}, {b, e}, {c, f}}

l[{{1, 2}, #]] & /@ Range[3]

{{a, d}, {b, e}, {c, f}}

{1, 2, 3} + {a, b, c}

{1 + a, 2 + b, 3 + c}

{1, 2, 3} == {a, b, c}

{1, 2, 3} == {a, b, c}

{1, 2, 3} == {a, b, c} // Thread

{1 == a, 2 == b, 3 == c}

```

```

glsys = {x + y, y - z, z + 2 x} == {1, 2, 3};
glsys // Thread

{x + y == 1, y - z == 2, 2 x + z == 3}

Solve[glsys, {x, y, z}]

{{x -> 4, y -> -3, z -> -5}}

```

■ Substitutionslisten

```

ClearAll["Global`*"]

gls = {x^2 + y == 3, y^2 + x == 3}

{x^2 + y == 3, x + y^2 == 3}

sol = Solve[gls, {x, y}]

{{x -> -1, y -> 2}, {x -> 2, y -> -1},
 {x ->  $\frac{1}{2}(-1 - \sqrt{13})$ , y ->  $\frac{1}{2}(-1 - \sqrt{13})$ }, {x ->  $-\frac{1}{2} + \frac{\sqrt{13}}{2}$ , y ->  $\frac{1}{2}(-1 + \sqrt{13})$ }}

Map[{x, y} /. # &, sol]

{{{-1, 2}, {2, -1}, { $\frac{1}{2}(-1 - \sqrt{13})$ ,  $\frac{1}{2}(-1 - \sqrt{13})$ }, { $-\frac{1}{2} + \frac{\sqrt{13}}{2}$ ,  $\frac{1}{2}(-1 + \sqrt{13})$ }}}

{x, y} /. sol

{{{-1, 2}, {2, -1}, { $\frac{1}{2}(-1 - \sqrt{13})$ ,  $\frac{1}{2}(-1 - \sqrt{13})$ }, { $-\frac{1}{2} + \frac{\sqrt{13}}{2}$ ,  $\frac{1}{2}(-1 + \sqrt{13})$ }}}

Map[Table[x^i + y^i, {i, 1, 4}] /. # &, sol] // Expand

{{{1, 5, 7, 17}, {1, 5, 7, 17}, {-1 -  $\sqrt{13}$ , 7 +  $\sqrt{13}$ , -10 - 4  $\sqrt{13}$ , 31 + 7  $\sqrt{13}$ },
 {-1 +  $\sqrt{13}$ , 7 -  $\sqrt{13}$ , -10 + 4  $\sqrt{13}$ , 31 - 7  $\sqrt{13}$ }}}

Table[x^i + y^i, {i, 1, 4}] /. sol // Expand

{{{1, 5, 7, 17}, {1, 5, 7, 17}, {-1 -  $\sqrt{13}$ , 7 +  $\sqrt{13}$ , -10 - 4  $\sqrt{13}$ , 31 + 7  $\sqrt{13}$ },
 {-1 +  $\sqrt{13}$ , 7 -  $\sqrt{13}$ , -10 + 4  $\sqrt{13}$ , 31 - 7  $\sqrt{13}$ }}}

```

■ Beispiel: Nullstellen eines Polynoms über \mathbb{Z}_p

```

ClearAll["Global`*"]

```



```

primeList = Select[Range[50], PrimeQ]

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47}

werteTafel = Table[{x, Mod[x^3 + x + 1, 31]}, {x, 0, 30}]

{{0, 1}, {1, 3}, {2, 11}, {3, 0}, {4, 7}, {5, 7}, {6, 6}, {7, 10},
 {8, 25}, {9, 26}, {10, 19}, {11, 10}, {12, 5}, {13, 10}, {14, 0}, {15, 12},
 {16, 21}, {17, 2}, {18, 23}, {19, 28}, {20, 23}, {21, 14}, {22, 7}, {23, 8},
 {24, 23}, {25, 27}, {26, 26}, {27, 26}, {28, 2}, {29, 22}, {30, 30}}

Select[werteTafel, #[[2]] == 0 &]

{{3, 0}, {14, 0}}

Map[#[[1]] &, %]

{3, 14}

Select[Range[0, 30], Mod[#^3 + # + 1, 31] == 0 &]

{3, 14}

sol[p_?PrimeQ] := Select[Range[0, p - 1], Mod[#^3 + # + 1, p] == 0 &]

? sol

Global`sol

sol[p_?PrimeQ] := Select[Range[0, p - 1], Mod[#1^3 + #1 + 1, p] == 0 &]

sol[13]

{7}

{#, sol[#]} & /@ primeList

{{2, {}}, {3, {1}}, {5, {}}, {7, {}}, {11, {2}}, {13, {7}}, {17, {11}}, {19, {}}, {23, {4}},
 {29, {26}}, {31, {3, 14}}, {37, {25}}, {41, {}}, {43, {38}}, {47, {25, 34, 35}}

{#, Factor[x^3 + x + 1, Modulus -> #]} & /@ primeList

{{2, 1 + x + x^3}, {3, (2 + x) (2 + x + x^2)}, {5, 1 + x + x^3},
 {7, 1 + x + x^3}, {11, (9 + x) (5 + 2 x + x^2)}, {13, (6 + x) (11 + 7 x + x^2)},
 {17, (6 + x) (3 + 11 x + x^2)}, {19, 1 + x + x^3}, {23, (19 + x) (17 + 4 x + x^2)},
 {29, (3 + x) (10 + 26 x + x^2)}, {31, (17 + x)^2 (28 + x)}, {37, (12 + x) (34 + 25 x + x^2)},
 {41, 1 + x + x^3}, {43, (5 + x) (26 + 38 x + x^2)}, {47, (12 + x) (13 + x) (22 + x)}}

```

■ Beispiel: Punkte auf einer elliptischen Kurve über \mathbb{Z}_p

```
ClearAll["Global`*"]
```

```

11 = Table[{x, y}, {x, 0, 10}, {y, 0, 10}];
Short[11, 4]

{{0, 0}, {0, 1}, {0, 2}, {0, 3}, {0, 4}, {0, 5}, {0, 6}, {0, 7}, {0, 8}, {0, 9}, {0, 10}},
<<9>>, {{10, 0}, {10, 1}, {10, 2}, {10, 3}, {10, 4},
{10, 5}, {10, 6}, {10, 7}, {10, 8}, {10, 9}, {10, 10}}}

12 = Join@@11;
Short[12, 5]

{{0, 0}, {0, 1}, {0, 2}, {0, 3}, {0, 4}, {0, 5}, {0, 6}, {0, 7}, {0, 8},
{0, 9}, {0, 10}, {1, 0}, {1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}, <<87>>,
{9, 5}, {9, 6}, {9, 7}, {9, 8}, {9, 9}, {9, 10}, {10, 0}, {10, 1}, {10, 2},
{10, 3}, {10, 4}, {10, 5}, {10, 6}, {10, 7}, {10, 8}, {10, 9}, {10, 10}}

ec11 = Select[12, Mod[#[[2]]^2 - #[[1]]^3 - 1, 11] == 0 &]

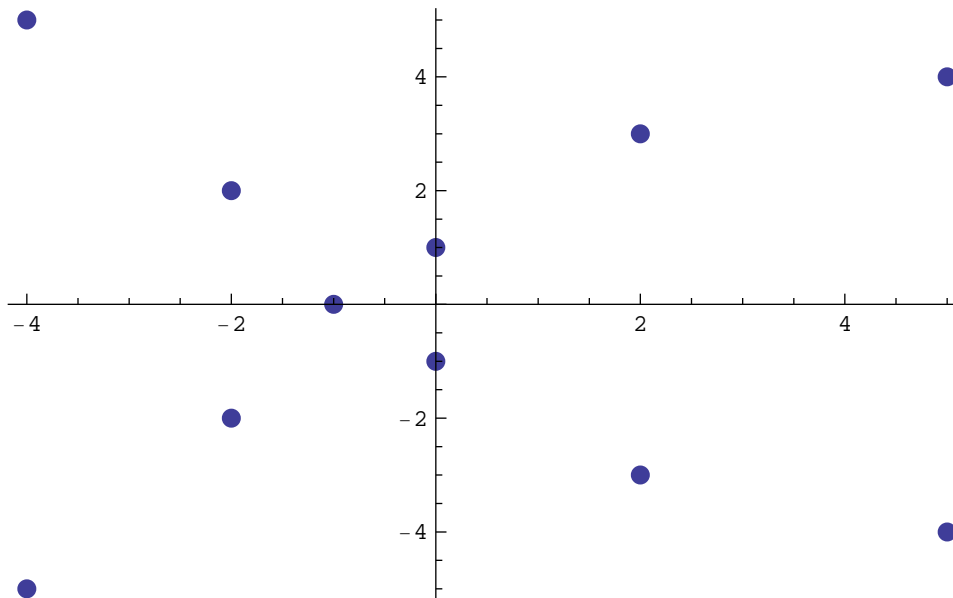
{{0, 1}, {0, 10}, {2, 3}, {2, 8}, {5, 4}, {5, 7}, {7, 5}, {7, 6}, {9, 2}, {9, 9}, {10, 0}}

ec11 = Select[Join@@Table[{x, y}, {x, -5, 5}, {y, -5, 5}], Mod[#[[2]]^2 - #[[1]]^3 - 1, 11] == 0 &]

{{-4, -5}, {-4, 5}, {-2, -2}, {-2, 2},
{-1, 0}, {0, -1}, {0, 1}, {2, -3}, {2, 3}, {5, -4}, {5, 4}}

ListPlot[ec11, PlotStyle -> PointSize[0.02]]

```



```

ec[p_?PrimeQ] :=
  Select[Join@@Table[{x, y}, {x, p}, {y, p}], Mod[#[[2]]^2 - #[[1]]^3 - 1, p] == 0 &]

primeList = Select[Range[50], PrimeQ]; Map[{#, Length[ec[#]]} &, primeList]

{{2, 2}, {3, 3}, {5, 5}, {7, 11}, {11, 11}, {13, 11}, {17, 17}, {19, 11},
{23, 23}, {29, 29}, {31, 35}, {37, 47}, {41, 41}, {43, 35}, {47, 47}}

```

■ Teillisten und Muster

■ Select, MatchQ und Cases

```

l = {1, a, b^c, d^3, e + f, sqrt[3]};

Cases[l, _-]

{b^c, d^3, sqrt[3]}

Select[l, MatchQ[#, _-] &]

{b^c, d^3, sqrt[3]}

Cases[l, x_ y_Integer]

{d^3}

Select[l, MatchQ[#, _Integer] &]

{d^3}

Cases[l, x_ y_Integer -> x^2 y]

{d^6}

Select[l, MatchQ[#, _Integer] &] /. x_ y_Integer -> x^2 y

{d^6}

DeleteCases[l, _-]

{1, a, e + f}

Select[l, ! MatchQ[#, _-] &]

{1, a, e + f}

u = 12 (1 + x)^5 // Expand
12 + 60 x + 120 x^2 + 120 x^3 + 60 x^4 + 12 x^5

Cases[u, 12 _-]

{12 x^5}

Cases[u, __ _]

{60 x, 120 x^2, 120 x^3, 60 x^4, 12 x^5}

```

```

Select[u, MatchQ[#, _ __] &]

60 x + 120 x2 + 120 x3 + 60 x4 + 12 x5

List @@ %

{60 x, 120 x2, 120 x3, 60 x4, 12 x5}

1

```

■ Weitere Listenkommandos

```

Cases[l, _Symbol]

{a}

Count[l, _Symbol]

1

MemberQ[l, _Times]

False

FreeQ[l, _Power]

False

FreeQ[l, _Times]

True

Count[u, _Times]

5

MemberQ[u, _Power]

False

FreeQ[u, _Power]

False

```

■ Zusammengesetzte Muster und Defaultmuster

```

l = {5, x2, ab+c, (a + b)c+d};

Clear[f];
f[x_] := x + 3;
Map[f, l]

{f[5], 3 + x, 3 + a, 3 + a + b}

```

```

Clear[f];
f[x : _] := x + 3;
Map[f, l]

{f[5], 3 + x2, 3 + ab+c, 3 + (a + b)c+d}

l = {1, x, x2, a, a x, a x2};

Cases[l, x-]

{x2}

Cases[l, x-.]

{x, x2}

Cases[l, _ x-]

{a x, a x2}

Cases[l, _ . x-]

{x, x2, a x, a x2}

```

■ Spezielle listenbasierte Datenstrukturen

■ Mengen

```

ClearAll["Global`*"]

l1 = {1, 2, 3, 4}; l2 = {6, 5, 4, 3}; l3 = {1, 3, 2, 1, 2};

l1 ∪ l2

{1, 2, 3, 4, 5, 6}

Union[l3]

{1, 2, 3}

l3 ∪ {}

{1, 2, 3}

l1 ∩ l2

{3, 4}

Complement[l1, l2]

{1, 2}

```

```
MemberQ[11, 1]
```

```
True
```

■ Vektoren und Matrizen

```
ClearAll["Global`*"]
```

```
VectorQ[v = {1, 2, 3}]
```

```
True
```

```
VectorQ[{{1}, {2, 3}}]
```

```
False
```

```
MatrixQ[{{1, 2}, {3, 4}}]
```

```
True
```

```
MatrixQ[{{1, 2}, {3, 4, 5}}]
```

```
False
```

```
m = {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}, {10, 11, 12}};
m // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{pmatrix}$$

```
m.v // MatrixForm
```

$$\begin{pmatrix} 14 \\ 32 \\ 50 \\ 68 \end{pmatrix}$$

```
v.v
```

```
14
```

```
Dimensions[m]
```

```
{4, 3}
```

```
(m1 = Transpose[m]) // MatrixForm
```

$$\begin{pmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{pmatrix}$$

```
m1 // Dimensions
```

```
{3, 4}
```

Weitere Beispiele

■ Komplexe Zahlen in Real- und Imaginärteil aufspalten

```

ClearAll["Global`*"]

l = Table[Random[Complex], {5}]

{0.0330212 + 0.465473 i, 0.155084 + 0.88999 i,
 0.821288 + 0.221346 i, 0.144917 + 0.0909742 i, 0.223424 + 0.8069 i}

Table[{Re[l[[i]], Im[l[[i]]]}, {i, Length[l]}]

{{0.0330212, 0.465473}, {0.155084, 0.88999},
 {0.821288, 0.221346}, {0.144917, 0.0909742}, {0.223424, 0.8069}}

Map[{Re[#], Im[#]} &, l]

{{0.0330212, 0.465473}, {0.155084, 0.88999},
 {0.821288, 0.221346}, {0.144917, 0.0909742}, {0.223424, 0.8069}}

{Re[#], Im[#]} & /@ l

{{0.0330212, 0.465473}, {0.155084, 0.88999},
 {0.821288, 0.221346}, {0.144917, 0.0909742}, {0.223424, 0.8069}}

{Re[l], Im[l]} // Thread

{{0.0330212, 0.465473}, {0.155084, 0.88999},
 {0.821288, 0.221346}, {0.144917, 0.0909742}, {0.223424, 0.8069}}

```

■ Komplexe Zahlen aus zwei Listen konstruieren

```

ClearAll["Global`*"]

l = {Table[Random[], {5}], Table[Random[], {5}]}

{{0.523902, 0.616014, 0.421656, 0.194263, 0.85307},
 {0.231122, 0.0517396, 0.461006, 0.957623, 0.667823}}

Table[l[[1, i]] + I l[[2, i]], {i, Length[l[[1]]]}]

{0.523902 + 0.231122 i, 0.616014 + 0.0517396 i,
 0.421656 + 0.461006 i, 0.194263 + 0.957623 i, 0.85307 + 0.667823 i}

#[[1]] + I #[[2]] & /@ Thread[l]

{0.523902 + 0.231122 i, 0.616014 + 0.0517396 i,
 0.421656 + 0.461006 i, 0.194263 + 0.957623 i, 0.85307 + 0.667823 i}

Map[Apply[#1 + I #2 &, #] &, Transpose[l]]

{0.523902 + 0.231122 i, 0.616014 + 0.0517396 i,
 0.421656 + 0.461006 i, 0.194263 + 0.957623 i, 0.85307 + 0.667823 i}

```

```

Thread[{l[[1]], l[[2]]} /. {x_, y_} -> x + I y

{0.523902 + 0.231122 i, 0.616014 + 0.0517396 i,
 0.421656 + 0.461006 i, 0.194263 + 0.957623 i, 0.85307 + 0.667823 i}

Apply[(#1 + I #2) &, l]

{0.523902 + 0.231122 i, 0.616014 + 0.0517396 i,
 0.421656 + 0.461006 i, 0.194263 + 0.957623 i, 0.85307 + 0.667823 i}

Thread[(#1 + I #2) &[l[[1]], l[[2]]]

{0.523902 + 0.231122 i, 0.616014 + 0.0517396 i,
 0.421656 + 0.461006 i, 0.194263 + 0.957623 i, 0.85307 + 0.667823 i}

(#1 + I #2) &[l[[1]], l[[2]]

{0.523902 + 0.231122 i, 0.616014 + 0.0517396 i,
 0.421656 + 0.461006 i, 0.194263 + 0.957623 i, 0.85307 + 0.667823 i}

l[[1]] + I l[[2]]

{0.523902 + 0.231122 i, 0.616014 + 0.0517396 i,
 0.421656 + 0.461006 i, 0.194263 + 0.957623 i, 0.85307 + 0.667823 i}

```

■ Gleitender Durchschnitt

```

ClearAll["Global`*"]

l = Table[Random[Integer, 20], {10}]

{17, 11, 3, 0, 1, 10, 11, 6, 11, 12}

sublists = Table[l[[i + Range[4]]], {i, 0, 6}]

{{17, 11, 3, 0}, {11, 3, 0, 1}, {3, 0, 1, 10},
 {0, 1, 10, 11}, {1, 10, 11, 6}, {10, 11, 6, 11}, {11, 6, 11, 12}}

Partition[l, 4, 1]

{{17, 11, 3, 0}, {11, 3, 0, 1}, {3, 0, 1, 10},
 {0, 1, 10, 11}, {1, 10, 11, 6}, {10, 11, 6, 11}, {11, 6, 11, 12}}

sums = Apply[Plus, #] & /@ sublists

{31, 15, 14, 22, 28, 38, 40}

Apply[Plus, sublists, {1}]

{31, 15, 14, 22, 28, 38, 40}

sums / 4

{31/4, 15/4, 7/2, 11/2, 7, 19/2, 10}

gleitenderDurchschnitt[l_List, n_Integer? (# > 0 &)] :=
  (Apply[Plus, #] & /@ Table[l[[i + Range[n]]], {i, 0, Length[l] - n}]) / n

```



```
l1 = Table[{x, 2 Sin[x] + Random[]}, {x, 0,  $\pi$ , 0.01}];
```

```
l2 = gleitenderDurchschnitt[l1, 20];
```

```
p1 = ListLinePlot[l1, AxesOrigin -> {0, 0}];  
p2 = ListLinePlot[l2, AxesOrigin -> {0, 0}];  
GraphicsGrid[{{p1, p2}}]
```

